

**Reply to “Comment on ‘The nature of slow dynamics in a minimal model of frustration-limited domains’” by J. Schmalian, P.G. Wolynes and S. Wu.** In a recent paper[1] we reported simulation results for a minimal model of frustration-limited domains with energy functional

$$\beta\mathcal{H}[\phi(\mathbf{r})] = \int d\mathbf{r} \left[ \frac{1}{2}\phi(\mathbf{r}) (\tau + k_0^{-2}(\nabla^2 + k_0^2)^2) \phi(\mathbf{r}) \frac{\lambda}{4!}\phi^4(\mathbf{r}) \right]. \quad (1)$$

We concluded that in all but a very small unexplored portion of the phase diagram the Langevin equation corresponding to Eq. 1,

$$\frac{\partial\phi(\mathbf{r})}{\partial t} = -\frac{\delta\mathcal{H}[\phi]}{\delta\phi(\mathbf{r})} + \eta(\mathbf{r}, t) \quad (2)$$

with Gaussian random force  $\eta(\mathbf{r}, t)$ , does not generate glassy dynamics. Although relaxation times do grow dramatically as a function of inverse temperature, we found no evidence for non-exponential multi-step relaxation. A simple Hartree approximation is in fact sufficient to capture the computed dynamics almost quantitatively. The validity of a harmonic reference system, we reasoned, demonstrates that activated processes and the possibility of an entropy crisis are not essential for dynamics at temperatures above (but not extremely close to) the fluctuation-induced first order transition temperature,  $\tau_{tr}$ . We further noted that facile growth of long-range order upon cooling below  $\tau_{tr}$  renders the study of a stationary, supercooled disordered state impossible. The relevance of our conclusions for the stripe glass scenario of Schmalian and Wolynes has recently been questioned by Schmalian *et al.*[2]. In particular, they point out that our simulations span a range of effective temperatures above that at which glassiness sets in,  $T_c$ , as predicted by either the self-consistent screening approximation (SCSA) or dynamical mean-field theory (DMFT).

First, we discuss the nature of the approximations used in [1] and [2] to calculate  $T_c$ . Schmalian *et al.* describe SCSA and DMFT as more sophisticated approaches than the mode-coupling approximation (MCA) analyzed in our work. Regardless of the level of sophistication of these approximations, we chose to focus on the MCA because it is the clear analog of Götze’s mode coupling theory (MCT) for relaxation of density fluctuations in liquids[3]. Although MCT overestimates  $T_c$  in model systems[4], simulations show a robust correlation between the predicted ergodic-nonergodic transition and the onset of caging in supercooled liquids[5, 6]. We found no hint of caging behavior in our simulations, even below the values of  $\tau$  for which the MCA predicts loss of ergodicity. Furthermore, the correspondence between static correlations and dynamical relaxation in our minimal model is much stronger than that exhibited by supercooled liquids, and the non-Arrhenius temperature

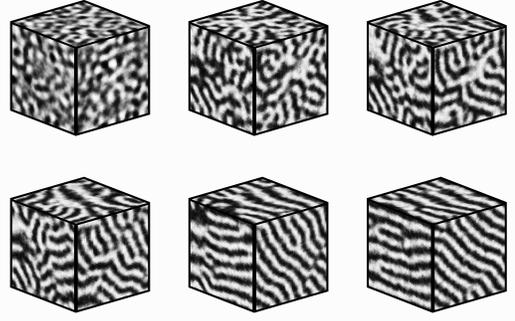


FIG. 1: Coarsening of a configuration whose structural correlations are consistent with the disordered solution of DMFT at  $\tau = -0.2$ ,  $q_0 = 0.5$ , and  $\lambda = 1$ . Field values  $\phi(\mathbf{r})$  at the surface of our (periodically replicated) simulation cell are shown for times  $t = 0$  (top left),  $t = 500$  (top middle),  $t = 1000$  (top right),  $t = 2000$  (bottom left),  $t = 5000$  (bottom middle), and  $t = 10,000$  (bottom right).

dependence of correlation times is coupled to exponential, rather than multi-step, relaxation. These prominent features of the dynamics generated by (2), while interesting, are therefore not generically similar to those found in supercooled liquids.

In the conclusion of our work we mentioned that growth of long-range lamellar order occurs even slightly below  $\tau_{tr}$ , indicating a lack of stable glassy behavior at temperatures below those studied thoroughly in [1]. Here, we address the criticism of Schmalian *et al.* by presenting detailed results for  $\tau < \tau_{tr}$ . For several disordered configurations representative of the equilibrium state at  $\tau = -0.14$ ,  $q_0 = 0.5$  and  $\lambda = 1$ , we have performed instantaneous quenches to  $\tau = -0.2$ , the temperature at which Schmalian *et al.* predict the onset of glassiness, and have followed the subsequent evolution in time. Instead of the aging behavior typical of a supercooled liquid [7] (weak time dependence of one-time quantities and relative *stability* of the disordered phase), coarsening occurs, i.e., the modulated order of the ground state rapidly appears in all regions of our simulation cell, and ordered domains gradually align. As a more explicit test of the stability of the glassy state described in [2], we have constructed initial configurations representative of the DMFT solution of Schmalian *et al.* for  $\tau = -0.2$ , drawing values for the Fourier components of  $\phi(\mathbf{r})$  from appropriate Gaussian distributions[8]. A typical trajectory originating at such a glassy state is depicted in Fig. 1. Again, the system steadily coarsens towards the ordered state. In Fig. 2 we show the peak of the structure factor,  $S_0(t)$ , as a function of time for examples of both types of initial conditions. After a short transient, the two results merge and grow approximately with the power law expected from previous studies of models in the Brazovskii class[9]. These low-temperature results suggest that the topography of (1) in this region of config-

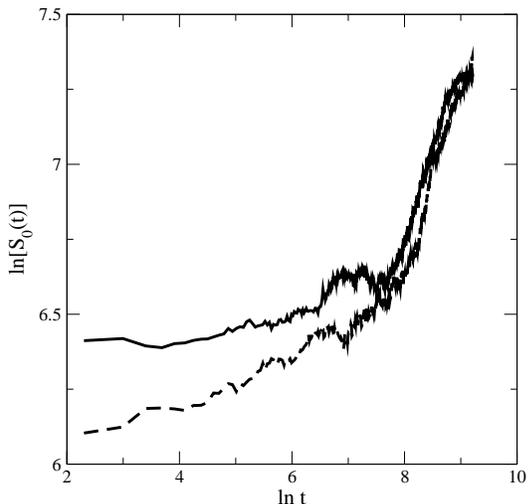


FIG. 2: Aging of the peak of the structure factor,  $S_0(t)$ , for two trajectories at  $\tau = -0.2$ ,  $q_0 = 0.5$ , and  $\lambda = 1$ . The dashed line corresponds to an initial configuration representative of equilibrium at  $\tau = -0.14$ . The solid line corresponds to an initial configuration consistent with the statistics of DMFT described in [2]. Although these results are anecdotal, late stage growth is consistent with a power law  $S_0(t) \propto t^\alpha$  with exponent  $\alpha \approx 0.5$ .

uration space differs from that envisioned by Schmalian, Wolynes and coworkers and that standard entropy crisis arguments must be modified in this context. In particular, the aging dynamics we have computed contrast strongly with the aging of a supercooled liquid, implying that the relevant region of the landscape (1) does not closely resemble that of a supercooled liquid at low temperatures[7, 10].

We do not deny that experiments in high temperature superconducting materials reveal slow wandering of stripes. This behavior may indeed be consistent with coarsening in two dimensions, which permits very slow rearrangement of modulated structures. In addition disorder and other factors neglected in (1) may also stabilize the quasi-disordered dynamics of hole-rich and hole-poor regions, perhaps giving rise to glassy dynamics rather than simple coarsening. One possibility

is that more complicated terms should be included in (1) that would stabilize the glassy state[11]. Another possibility is that a stable glassy state arises only in the limit of strong coupling ( $\lambda \rightarrow \infty$ ). Investigation of these possibilities is certainly a worthy direction for future research.

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